Informacije (MIDEM

Journal of Microelectronics, Electronic Components and Materials Vol. 48, No. 1(2018), 63 – 70

A Novel Approach to Reduce the PMEPR of MCPC Signal Using Random Phase Algorithm

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Abstract: This paper aims to reduce the Peak-to-Mean Envelope Power Ratio (PMEPR) of a Multicarrier Complementary Phase Coded (MCPC) signal. A MCPC signal consists of P subcarriers which are phase modulated by N distinct phase sequences. Each of these P subcarriers is spaced by the inverse duration of a phase element, which constitutes an Orthogonal Frequency Division Multiplexing (OFDM) signal. A probabilistic approach, namely, Random Phase Updating (RPU) algorithm, is used to reduce the PMEPR of the generated MCPC signal. The technique is applied to higher order MCPC signals and a comparison of the peak sidelobe ratio (PSLR) and integrated sidelobe ratio (ISLR) is performed. The complex envelopes, autocorrelations and ambiguity functions of the MCPC signal obtained by the above mentioned methods are analysed. The Complementary Cumulative Distribution Function (CCDF) is plotted to validate the PMEPR reduction obtained by the application of the RPU algorithm which enables us to determine the most suitable approach required for radar applications.

Keywords: Integrated Sidelobe Ratio (ISLR); Multicarrier Complementary Phase Coded (MCPC); Orthogonal Frequency Division Multiplexing (OFDM); Peak to Mean Envelope Power Ratio (PMEPR); Peak Sidelobe Ratio (PSLR); Random Phase Updating (RPU)

Nov način zniževanja PMEPR MCPC signala z uporabo naključnega faznega algoritma

Izvleček: Članek opisuje zmanjšanje vršno-srednjega razmerja moči (PMEPR) večnosilčnega komplementarno fazno kodiranega (MCPC) signala. MCPC signal vsebuje podnosilce P, ki so fazno modulirani z N različnimi faznimi sekvencami. Vsak podnosilce P je ločen z inverznim trajanjem faznega elementa, ki oblikuje OFDM signal. Za zniževanje PMEPR je uporabljen verjetnostni pristop z naključno fazno osvežitvijo (RPU). Tehnika je uporabljena na višjih redih MCPC signala. Opravljanje primerjava razmerja vrhnjega snopa (PSLR) in razmerja integriranega snopa (ISLR). Analizirani so kompleksni ovoji, avtokorelacije in nejasne funkcije MCPC signala. Za validacijo znižanja PMEPR na osnovi RPU funkcije je uporabljena CCDF funkcija kot najboljši pristop za uporabo v radarju.

Ključne besede: ISLR; MCPC; OFDM; PMEPR; PSLR; RPU

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1 Introduction

The most important characteristics of a radar signal are its range and resolution [1]. In order to improve the range of the signal, the pulse width must be increased. This hampers its resolution. On the other hand, decreasing the pulse width improves the resolution of the radar signal but results in deterioration of its range. We use pulse compression technique to balance the tradeoff between the range and resolution of the radar signal. Phase coding of the transmitted radar signal helps achieve pulse compression. The advantage of a multicarrier system over single carrier transmission in terms of bandwidth efficiency [2] is clearly demonstrated by the Orthogonal Frequency Division Multiplexing (OFDM) technique. OFDM technology forms the foundation for a number of communication systems such as Digital Audio and Video Broadcasting, IEEE 802.11g, Digital Subscriber Lines (xDSL). The latest applications include LTE and LTE Advanced. OFDM has also been applied to radar systems for object tracking and target detection. This application has been realized in different types of multipath and clutter environments. However, the multicarrier signals have high variations present in the complex envelope. These variations are quantified by a parameter, namely Peak to Mean Envelope Power Ratio (PMEPR). Higher value of PMEPR indicates more abrupt variations in the complex envelope of the signal and the power amplifier at the transmitter end has to be very sensitive to track these sudden variations. Since design of such a sensitive amplifier is complicated, reduction of the PMEPR of the radar signal becomes essential.

The radar signal is phase coded using P4 [3] phase sequences which are complementary in nature. This helps us to accomplish Pulse Compression. The generated signal is a MCPC Signal as described by N. Levanon in [4]. The only drawback of this signal is its high value of PMEPR.

Several attempts have been made to reduce the effect of PMEPR in multicarrier schemes and emphasis is on data transmission applications, using methods such as near- complementary sequence [5], peak power reduction of OFDM signals with sign adjustment [6], tone reservation [7,8] and a joint technique [9]. However several authors have investigated to reduce PMEPR in multicarrier signals for radar applications. In [10], phase modulation is used and in [11] PMEPR is reduced using iterative least square algorithm and in [12] genetic algorithm used. The objective of this paper is to address the issue of high PMEPR of a MCPC radar signal using RPU algorithm whose implementation until now has only been restricted to data transmission systems.

2 Characteristics of MCPC Signal

The multicarrier phase-coded signal is based on the principle of OFDM technique. It comprises of N subcarriers which are phase modulated by N distinct phase sequences. The frequencies of the subcarriers are $1/t_{\rm b}$ apart, where $t_{\rm b}$ is the duration of each phase element.

The phase sequences are generated using P4 phase sequences.

The equation for generating P4 phase sequence is given in equation 1.

$$\phi_q = \frac{\pi}{N} (q-1)^2 - \pi (q-1) \quad q = 1, 2, 3 \dots N$$
 (1)

For a 5 x 5 MCPC we generate the P4 phase sequences by setting N = 5. The first sequence which is obtained by cyclically shifting to attain the other 4 phase sequences. The P4 phase sequences obtained are shown in Table 1. All the phases are in radians.

Table 1: P4 Phase Sequences

Seq 1 [rad]	Seq 2 [rad]	Seq 3 [rad]	Seq 4 [rad]	Seq 5 [rad]
0	-2.513	-3.769	-3.769	-2.513
-2.513	-3.769	-3.769	-2.513	0
-3.769	-3.769	-2.513	0	-2.513
-3.769	-2.513	0	-2.513	-3.769
-2.513	0	-2.513	-3.769	-3.769

The phase sequence order of a MCPC signal is used to indicate the phase sequence which is used to modulate a particular subcarrier. For example, a phase sequence order of [3 5 2 1 4] involves the phase modulation of the first subcarrier with phase sequence 3, second subcarrier with phase sequence 5 and so on, where the phase sequences are obtained from Table 1.The complex envelope [3] of the MCPC signal is given by equation 2.

Using the above equations the complex envelopes for MCPC signals having different number of subcarriers such as 7×7 , 9×9 , 11×11 , etc. can be generated using their respective phase sequences. The block diagram for generating the MCPC signal is as shown in Fig. 1.

The PMEPR value for different phase sequence orders is

$$s(t) = \begin{cases} \sum_{p=1}^{N} A_p \exp\left[j\left\{2\pi f_s t\left(\frac{N+1}{2} - p\right) + \theta_p\right\}\right] \sum_{q=1}^{N} u_{p,q} \left[t - (q-1)t_b\right], & 0 \le t \le Nt_b \\ 0, & elsewhere \end{cases}$$
(2)

Where, $u_{p,q}(t) = \begin{cases} \exp(j\phi_{p,q}), & 0 \le t \le t_b \\ 0, & elsewhere \end{cases}$

 $A_{\rm p}$ is the amplitude weight applied to the subcarriers and $\theta_{\rm p}$ is the random phase shift introduced by the transmitter to each carrier. $\phi_{\rm p,q}$ is the qth phase of the pth subcarrier.



Figure 1: Generation of MCPC Signal

illustrated in Table 2.

Table 2: PMEPR values of MCPC signals for different sequence orders

Sequence order	PMEPR using P4
[3 5 2 1 4]	4.39
[3 4 5 1 2]	1.73
[3 1 2 5 4]	2.97
[3 2 4 1 5]	3.48

The ambiguity function for the phase sequence order [3 5 2 1 4] is depicted in Fig. 2.



Figure 2: Ambiguity Function of MCPC signal

Autocorrelation function is the correlation of a signal with a delayed copy of itself as a function of delay [13]. The width of the mainlobe gives an idea about the range of the radar signal and the sidelobe power levels govern the resolution of the signal.

Ambiguity function [14] is a two-dimensional function of delay and Doppler frequency that measures the correlation between a waveform and its Doppler distorted version. Autocorrelation and the ambiguity function together help analyze the target detection capabilities of the radar signal. When we have multiple point targets we have a superposition of ambiguity functions. A weak target located near a strong target can be masked by the sidelobes of the ambiguity function cantered around the strong target. Hence, we have to minimize the minor lobes for detection of secondary targets.

The quality of the radar signal can also be assessed using Peak Sidelobe Ratio (PSLR) and Integrated Sidelobe Ratio (ISLR). The Peak Sidelobe Ratio (PSLR) is the ratio between the returned signal of the mainlobe and that of the maximum sidelobe power. The Integrated Sidelobe Ratio (ISLR) is the ratio of the energy in the sidelobes to that contained in the mainlobe. The PSLR and ISLR for the conventional 5 x 5 MCPC signal were found to be 8.32dB and 3.34dB respectively.

3 Random Phase Updating Algorithm

The only drawback of the MCPC signal is its high value of PMEPR. Reducing this quantity will result in the reduction of the variations in the complex envelope. This issue can be addressed by using one of the methods suggested in [5]. However the technique thus adopted must not only ensure a reduction in PMEPR but also maintain acceptable autocorrelation and ambiguity functions. An effective approach is to make use of the Random Phase Updating (RPU) algorithm [15] which comes under the purview of the probabilistic domain. The block diagram for generating the MCPC signal with RPU algorithm is shown in Fig. 3.



Figure 3: RPU Algorithm for generation of MCPC signal

The random phase updating algorithm generates phases and adds them to the pre-existing P4 phase values as given by equation 3.

$$\left(\phi_{p}\right)_{i} = \left(\phi_{p}\right)_{i-1} + \left(\Delta\phi_{p}\right)_{i}$$
(3)

In equation 3, *i* denotes the iteration, and *p* denotes the subcarrier. $(\phi_{p'_i})_i$ is the phase of the p^{th} subcarrier in the i^{th} iteration and $(\Delta \phi_p)_i$ is the incremental phase added to the p^{th} subcarrier in the i^{th} iteration.

The algorithm uses the number of iterations as the control parameter. The incremental phases are generated based on a particular probability density function and added to each subcarrier. Gaussian distribution or uniform distributions are used to generate these incremental phases. The complex envelope is obtained and the corresponding value of PMEPR is calculated for every iteration. Once the required number of iterations is carried out, the complex envelope and the phase sequences corresponding to the lowest value of PMEPR are selected. The autocorrelation function and the ambiguity function are plotted for the selected complex envelope. The flowchart in Fig. 4 describes the random phase updating algorithm.



Figure 4: Flowchart of the RPU Algorithm

The Gaussian distribution is given by $\Delta \phi_n = N(0, x^2)$

The Uniform distribution is given by $\Delta \phi_p = Unif(0, x^2)$

Here, $(\Delta \phi_p)$ is the incremental phase generated based on a particular distribution. *x* belongs to {0.1, 0.25, 0.5, 0.75, 1} for a 5 x 5 MCPC signal. Similarly, *x* belongs to {0.1, 0.25, 0.4, 0.55, 0.7, 0.85, 1} for a 7 x 7 MCPC signal and {0.1, 0.21, 0.32, 0.43, 0.55, 0.66, 0.77.0.88, 1} for a 9 x 9 MCPC signal. For each subcarrier, the incremental phases are obtained by calculating the CDF of one of the values in the vector '*x*' which is selected randomly.

4 Results

In this section, a comparison is made between the conventional MCPC signal and the signal subjected to the Random Phase Updating Algorithm for a large number of iterations. This technique has been applied to the MCPC signal that is based on the cyclic shifts of the P4 phase sequences for the order [3 5 2 1 4]. The complex envelope, autocorrelation and ambiguity function obtained using the RPU algorithm are plotted against those obtained using the conventional method.

In the random phase updating algorithm, the random numbers generated can be either repetitive or non-repetitive in nature. If the random numbers are repetitive, the number of possible combinations is large. For a 5 x 5 MCPC signal, there are 5^5 different combinations possible if the random numbers are repetitive and only 5! combinations if the random numbers are non-repetitive. A comparison of the results obtained using both the results is made in this section.

Further, for the generation of the incremental phases, the random phase updating algorithm uses either Gaussian Distribution or Uniform Distribution. A comparison of the results obtained using the above mentioned distributions along with the two methods of generation of random numbers is performed in this section.

Due to the random nature of the phase updating process, the complex envelope, autocorrelation function and ambiguity function need not be unique. However, the lowest value of PMEPR for the complex envelope remains the same when the number of iterations are very large.

It could be observed that the lowest value of PMEPR obtained when the random numbers were generated in a repetitive manner was almost identical to those obtained by generating non-repetitive numbers.

4.1 RPU Using Gaussian Distribution

The results obtained in this subsection illustrate the complex envelope, autocorrelation function and the



Figure 5: Complex Envelope of MCPC signal using RPU algorithm

ambiguity function obtained for a 5 x 5 MCPC signal with phase sequence [3 5 2 1 4] using the RPU algorithm where the incremental phases are generated based on Gaussian distribution. Fig. 5, Fig. 6 and Fig. 7 illustrate the case where the random numbers are non-repetitive in nature.



Figure 6: Autocorrelation Function of MCPC Signal using RPU Algorithm



Figure 7: Ambiguity Function of MCPC Signal using RPU Algorithm

Table 3 shows the comparison of PMEPR between conventional method and the RPU algorithm.



Absolute Value

Figure 8: Complex Envelope of MCPC signal obtained by RPU Algorithm

It can be clearly observed that the PMEPR values obtained using both the methods of generating random numbers are identical and better than those obtained using the conventional method.

The autocorrelation function shown in Fig. 6 has sidelobe power levels at approximately 15dB. This shows that the target detection capabilities of the radar signal are preserved after applying the technique.

From Fig. 7 it can be seen that the sidelobe ridges in the ambiguity function are lower for high Doppler shifts when compared to the conventional MCPC signal demonstrating that the target detection capabilities have been conserved.

The PSLR and ISLR for the MCPC signal after the application of the RPU technique were found to be -6.92dB and 4.45dB respectively. It can be observed that these values are higher than that obtained for the conventional MCPC signal, showing that there is a slight degradation in the resolution of the signal. There is a trade-off between PMEPR reduction and increased sidelobe-power levels. However, this minor disadvantage of distribution of the mainlobe power amongst the sidelobes does not compare with the advantage of PMEPR reduction.

Sequence Order	PMEPR using conventional MCPC Signal	PMEPR using RPU algorithm		
		Using non-repetitive random numbers	Using repetitive random numbers	
[3 5 2 1 4]	4.39	2.99	2.99	
[3 4 5 1 2]	1.73	1.53	1.54	
[3 1 2 5 4]	2.97	2.59	2.58	
[3 2 4 1 5]	3.48	2.27	2.25	

Table 3: PMEPR comparison table



Figure 9: Autocorrelation Function of MCPC Signal obtained by RPU Algorithm

4.2 Using Uniform Distribution

This section demonstrates results obtained when the incremental phases are generated based on Uniform distribution for the sequence order [3 5 2 1 4]. The graphs plotted in Fig. 8, Fig. 9 and Fig. 10 illustrate the complex envelope, autocorrelation function and ambiguity function respectively for this case.

The PMEPR comparison between conventional MCPC signal and the signal obtained by the application of RPU algorithm based on Uniform distribution is illustrated in Table 4.



Figure 10: Ambiguity Function of MCPC Signal obtained by RPU Algorithm



Figure 11: CCDF comparison

Sequence	PMEPR using Conventional MCPC Signal	PMEPR using RPU algorithm	
Örder		Using non-repetitive random numbers	Using repetitive random numbers
[3 5 2 1 4]	4.39	3.01	2.99
[3 4 5 1 2]	1.73	1.57	1.53
[3 1 2 5 4]	2.97	2.56	2.60
[3 2 4 1 5]	3.48	2.24	2.26

From Table 4, it can be noted that the PMEPR value has considerably reduced for all sequence orders when RPU algorithm is incorporated in the phase generation process of MCPC signal generation.

The autocorrelation function obtained indicates peak sidelobe power levels to be approximately 10dB which suggests that the target tracking ability of the signal based on Uniform distribution is marginally inferior to the signal obtained using Gaussian distribution.

The ambiguity function obtained shows that the signal has low sidelobe power levels at higher Doppler shifts similar to the case when Gaussian distribution is used, which is a favourable aspect. The PSLR and ISLR were found to be 4.75dB and 7.19dB respectively. The resolution of the radar signal is worse than that of the conventional MCPC signal and that of the signal obtained using Gaussian distribution but is still effective in reducing PMEPR.

4.3 Complementary Cumulative Distribution Function (CCDF)

The CCDF curve provides an idea of the distribution of power of the complex envelope around the mean. It is

Table 4: PMEPR comparison table

a plot of Power levels above the Average Power in dB vs Probability of occurrence of that particular power level above the mean power in the complex envelope under consideration. As the area under the curve increases, the power variation around the mean increase and this leads to an increased value of PMEPR. Conversely, as the area under the curve reduces, the PMEPR also has a lower value as the power variations around the mean is reduced.

The graph in Fig. 11 illustrates the comparison between the conventional MCPC signal and the signals obtained using the RPU algorithm with Gaussian distribution. It can clearly be seen that the complex envelope obtained using the RPU algorithm with Gaussian PDF has a much lesser area than the conventional MCPC signal and hence possesses a much lesser value of PMEPR. Thus the results obtained using the CCDF graph are in coherence with those shown in Table 3 and Table 4.

4.4 RPU Algorithm applied to Higher Order MCPC Signals

In this subsection, the RPU algorithm is applied to MCPC signals with greater number of subcarriers to assess whether the technique performs favourably in different scenarios. From the previous subsections, we can observe that the PMEPR value obtained using both Gaussian distribution and Uniform distributions are identical. Therefore, either of these distributions can be used for the reduction of PMEPR. Table 5 and Table 6 illustrate the PMEPR comparison between conventional MCPC and MCPC signal obtained using RPU algorithm for a 7 x 7 and 9 x 9 MCPC signal respectively. The number of possible sequence orders for a 7 x 7 and a 9 x 9 MCPC signal are 7! and 9! respectively. Since the values are very large, the Table 5 and Table 6 shows the sequence orders corresponding to the highest, lowest and an intermediate value of PMEPR obtained for a given order of the MCPC signal.

Table 5: PMEPR comparison for MCPC of order 7 x7

Sequence Order 7 x 7	PMEPR using Conventional MCPC signal	PMEPR using RPU algorithm
[2567413]	6.14	3.44
[7 1 2 3 4 5 6]	1.92	1.75
[7 1 3 2 6 4 5]	4.01	3.29

It can be inferred that the RPU algorithm delivers promising results in terms of PMEPR reduction for a 7x7 and 9x9 MCPC signal and can be suitably applied to higher order signals.

Table 6: PMEPR comparison for MCPC of order 9 x 9

Sequence Order 9 x 9	PMEPR using Conventional MCPC signal	PMEPR using RPU algorithm
[591724368]	7.76	3.43
[567891234]	1.95	1.57
[597123684]	4.86	3.56

The PSLR and ISLR for the discussed signals are shown in Table 7.

Table 7: PSLR and ISLR comparison table

Signal	PSLR(dB)	ISLR(dB)
7 x 7 Conventional MCPC	-7.38	5.52
7 x 7 MCPC with RPU	-7.35	6.55
9 x 9 Conventional MCPC	-7.33	7.04
9 x 9 MCPC with RPU	-6.89	7.43

It can be seen that in both 7 x7 and 9 x 9 MCPC signals, the conventional MCPC signal has lower PSLR and ISLR values than that obtained after application of the RPU technique. This shows that the resolution degrades and follows the trend of the 5 x 5 case. The advantage of PMEPR reduction compensates this limitation.

5 Conclusion

The MCPC signal has many advantages in terms of bandwidth efficiency and pulse compression capability when compared to other radar signals which makes it more suitable for radar applications. Its only limitation is the high value of PMEPR. This paper has successfully addressed this drawback through the application of the random phase updating algorithm.

Section IV showed the application of the RPU algorithm based on Gaussian and Uniform distribution and both techniques provided favourable results. The technique was also found to be successful in reducing PMEPR for higher order MCPC signals as well. The CCDF further validates the reduction of PMEPR by portraying the power distribution about the mean.

The autocorrelation functions plotted for the complex envelopes generated using Gaussian and Uniform distribution indicate that the sidelobe levels using Gaussian distribution is lesser than that of the Uniform distribution. Though the PMEPR values obtained for a particular phase sequence is identical in both these distributions, the Gaussian distribution fares slightly better in resolving the targets due to a lower sidelobe power level. The Random Phase Updating algorithm being an iterative approach is computationally intensive and increases design complexity of the radar system. The random nature of the procedure makes in-depth analysis of the technique difficult. However, the advantages of this technique dominate these limitations and can be considered as a successful approach to reduce PMEPR, aiding the generation of a better MCPC signal.

6 Acknowledgment

The authors are grateful to Department of Electronics and Communication, Ramaiah Institute of Technology, Bangalore

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Arrived: 09. 01. 2018 Accepted: 27. 03. 2018