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Harmonic modeling of full-wave diode rectifier for nonuniform load currents

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Abstract: This paper proposes an equivalent circuit model and formulation for presentation of characteristic harmonic components generated by full wave diode rectifiers for nonuniform load currents. In conventional equivalent circuits, only the uniform current harmonics on AC side of converters are considered. For an exact and general analysis, the proposed model involves both nonuniform current harmonics on AC side and voltage harmonics on DC side of the rectifiers. The obtained expressions are valid for a general load state without simplifying. The proposed model converts the full-wave diode rectifier into a linear circuit with regards to harmonics. The model depends on Fourier series expansion for the load voltage and the source current waveforms. Simulation results and exprerimental results validate the proposed model and the given expressions

Keywords: Equivent circuit, harmonic model, nonuniform load current, rectifier.

Harmonično modeliranje polnovalnega diodnega usmernika za nekonstantne bremenske tokove

Izvleček: Članek predlaga model ekvivalentnega vezja za predstavitev karakterističnih harmonskih komponent, ki jih proizvaja polnovalni diodni usmernik za nekonstantne bremenske tokove. V običajnih ekvivalentnih vezjih so običajno upoštevani le konstantni tokovi na AC strani. Predlagan model, za natančno analizo, vključuje tako nekonstantne tokovne harmonike na AC strani kakor tudi napetostne harmonike na DC strani usmernika. Tako dobljeni izrazi veljajo za splošno breme brez poenostavitev. Predlagan model pretvori polnovalen diodni usmernik v linearno vezje z upoštevanjem harmonikov. Temelji na razvoju Fourierove vrste za bremensko napetost in valovnem vhodnem toku. Simulacijski rezultati in poizkusi potrjujejo predlagan model.

Ključne besede: Nadomestno vezje, harmonski model, nekoonstanten tok, diode

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1 Introduction

The purpose of harmonic studies is to quantify the distortion in voltage and/or current waveforms at various locations in a system. The need for a harmonic study may be indicated by excessive measured distortion in existing systems or by installation of harmonic producing equipment. One important step in harmonic studies is to characterize and to model harmonic-generating sources.

In general, major harmonic sources can be categorized as (1) Devices that generate harmonics during their switching processes. The most commonly seen are power electronic devices, (2) Devices that generate harmonics due to their nonlinear voltage-current characteristics [1-3] such as transformers, reactors, ac arc furnaces; (3) Hybrid devices that include both types of aforementioned devices such as DC arc furnaces and fluorescent lamps with electronic ballasts [4-5]; and (4) Devices such as rotating machines that harmonics are generated because of nonsinusoidal flux distribution in the stator and the harmonic interaction between the stator and field windings [6].

The AC/DC converters are one of the most important harmonic sources in electrical systems. Accurate harmonic analysis of AC/DC converters has got more attention in the last decades. Many harmonic models have been proposed for representing power electronic devices [7-10]. The most common model is in the form of a harmonic current source, which is specified by its magnitude and phase spectrum. Three basic approaches used to build detailed models include developing analytical formulae for the Fourier series as a function of terminal voltage and operating parameters for the harmonic source, developing analytical models for harmonic source operation and solving for its current waveform by a suitable iterative method, and solving for harmonic source steady-state current waveform with time-domain simulation. A more practical approach for the harmonic analysis of an electrical system containing many converters is to use time domain simulation [11]. The interaction between the AC and DC side harmonics in a converter system must be correctly represented.

A time domain sampled-data model method for the computation of the ac current and dc voltage harmonic generated by a capacitor filtered three-phase uncontrolled rectifier is presented in [12]. The approach employs numerical iteration to determine the diode's turn-on and turn-off times and thereby determine the circuit's steady state solution. In [13], harmonic currents of three phase bridge uncontrolled rectifier are analyzed in both continuous domain and discrete domain. Moreover, the further analysis on boundary condition for harmonic current is put forward and generalized. Modeling and simulation of 6-pulse and 12-pulse rectifiers with impacts to input current harmonics are given in [14]-[15]. The paper [16] develops a systematic state-space approach to the modeling of boost type ac–dc converters and reduction of output ripples. The bridge rectifier voltage is modeled as a periodic disturbance whose harmonics have given frequencies but uncertain phases and magnitudes.

A possible alternative to time domain simulation is frequency domain methods [17-19]. Noncharacteristic line current harmonics of AC/DC converters with high dc ripple are investigated by assumption of negligible small commutation in [20]. By means of frequency domain analysis, analytical models are derived based on the first terms of the series expansion of the switching function. A MATLAB-based method to calculate the harmonic amplitudes of rectifier's DC-side output current is given in [21].

In primary investigations of harmonics relating to converters, the current on DC side of the converter systems was assumed uniform that resulted rectangular current in AC side of the converter. In this approach, the order and magnitude of harmonics could be calculated simply. Although considering more details about converter structure and possible practical conditions causes complexity in harmonic analysis, it results in real and accurate converter harmonics spectrum. Therefore, different methods are required to analyze current converter accurately. Using transfer functions or switching function, solving state equations of system, converter models in frequency domain, and solving time domain equations of the converter numerically are samples of these investigations [18, 22-23]. For general and accurate analysis, nonuniform structures of currents on DC and AC sides of converter systems must be considered.

In this study, the exact electrical equivalent circuit and expressions of AC/DC converters with respect to harmonics are introduced. Although the AC/DC converters are switching circuits, the method converts the switching circuit into a linear circuit with regards to harmonics. The paper is organized as follows. First, the conventional equivalent circuits of the rectifiers are pointed out, then a new equivalent circuit with respect to harmonic currents on the AC side and output voltage harmonics on DC side of the rectifiers is proposed. The model and expressions are developed for non-constant valued/nonuniform load currents.

2 Conventional equivalent circuit of full-wave rectifiers

Converters or rectifiers using semiconductor switching devices generate harmonics caused by the behavior of switching. The circuit configuration of a full-wave diode rectifier, of which the equivalent circuit will be described in detail, is shown in Fig.1. The diodes are assumed to be ideal.

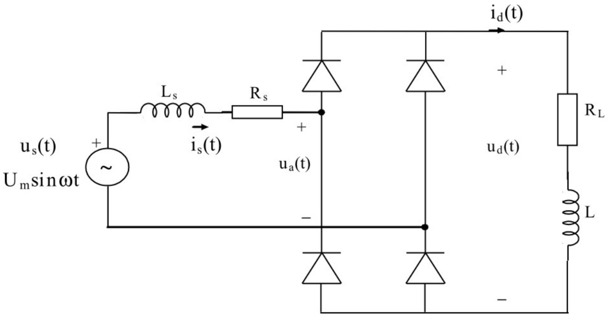


Figure 1: Configuration of a full wave diode rectifier

In a full wave diode rectifier, the output voltage, $u_d(t)$, is not dependent on the load in case that the equivalent serial resistance and inductance of the source are zero. The source current, $i_s(t)$ has a bidirectional form of the load current, $i_d(t)$. In conventional equivalent circuits, the amplitude of harmonic currents generated by a diode rectifier is considered to be constant by using a smoothing reactor on the DC side. For the condition of constant/uniform load current, in conventional equivalent circuits, the rectifier has been widely considered as an ideal current source for harmonics as follows

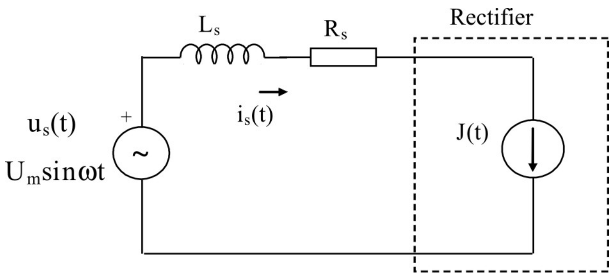
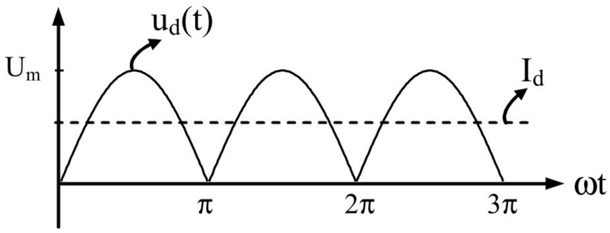
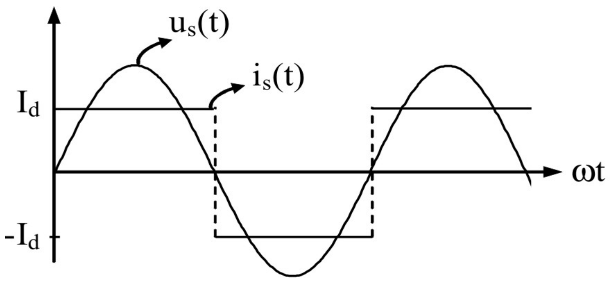


Figure 2: Conventional equivalent circuit to $i_s(t)$ on AC side

With regard to this condition, the load voltage/current and the source voltage/current of Fig.1 are available in many textbooks [24] and also given in Fig.3.



(a) Load voltage and current



(b) Source voltage and current

Figure 3: Waveforms relating to conventional situation

In Fig.1 and Fig.2, the resistor R_s and inductor L_s represent the source internal impedance. Relating to a uniform load current, the average values of the load voltage and load current in Fig. 3 are given as follows.

$$U_d = \frac{2U_m}{\pi} \quad (1.a)$$

$$I_d = \frac{U_d}{R_L} \quad (1.b)$$

3 Proposed equivalent circuit and formulation for nonuniform load currents

In conventional equivalent circuits of rectifiers, for constant valued/uniform load currents, current harmonics on AC side are considered. Whereas, in general situation, load and source currents are not always uniform as shown in Fig.4, in steady state. Moreover, in rectifier circuits, there are both current harmonics on AC side and voltage harmonics on DC side. For an exact and general analysis, it is necessary to deal with both harmonics not having uniform structure. For this purpose, a new equivalent circuit model containing both nonuniform current harmonics and voltage harmonics is proposed. The model depends on Fourier series expansion for the load voltage and the source current waveforms.

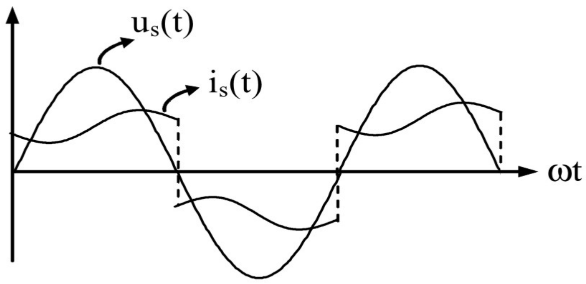


Figure 4: Waveforms relating to nonuniform current situation

First, Let's consider the output voltage harmonics in DC side of the rectifier circuit in Fig.1. In the rectifier circuits, an AC source is processed through a set of switches to create a well-defined waveform. We can represent the combined action of an actual source (AC source) and a set of switches by an equivalent source. The equivalent source provides a very strong advantage: The new circuits are linear, and avoid the nonlinearity and complication of switches. We can use superposition, Laplace transforms, or other techniques from linear network analysis to analyze rectifiers. Based on superposition, a term-by-term for the Fourier series of the current and voltage in the rectifier circuits can be solved. Equivalent voltage source, $u_d(t)$, applied to the load in Fig.1, is presented in Fig.5.

The equivalent voltage source, $u_d(t)$, contains both the fundamental component and harmonic components of the load voltage. The waveform of equivalent source, thereby the output voltage of the rectifier circuit, is given in Fig.3.a. The Fourier series for the voltage $u_d(t)$ can be expressed in trigonometric form as

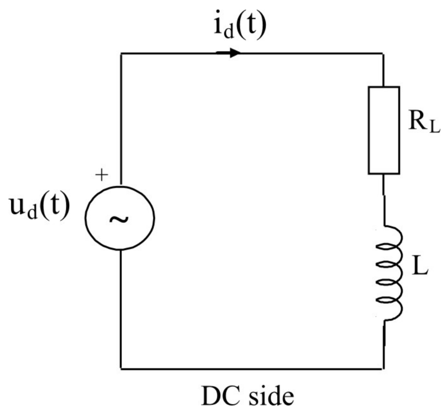


Figure 5: Equivalent voltage source, $u_d(t)$, applied to the load

$$u_d(t) = \frac{2U_m}{\pi} - \sum_{n=2,4,6...}^{\infty} \frac{2U_m}{\pi}\left(\frac{2}{n^2-1}\right)\cos(n\omega t) \quad (2)$$

Every component of Fourier series corresponds to a voltage source. They are symbolized as U_d, $u_{d2}(t)$, $u_{d4}(t)$, in Fig.6, where U_d is average load voltage as DC source, $u_{d2}(t)$ is a AC source having ω_2 frequency, $u_{d4}(t)$ is a AC source having ω_4 frequency,

Second, Let's deal with the current harmonics on AC side of the rectifier circuit in Fig.1. For this purpose, first, the load current on DC side must be defined. For general situation, the nonuniform load current, $i_d(t)$, is expressed by Fourier series as follows,

$$i_d(t) = \frac{2U_m}{\pi}\left\{\frac{1}{R_L} - \sum_{n=2,4,6...}^{\infty}\left(\frac{2}{n^2-1}\right)\left(\frac{1}{\sqrt{{R_L}^2+(n\omega L)^2}}\right)\cos(n\omega t-\varphi_n)\right\} \quad (3)$$

where, $\varphi_n = \tan^{-1}\frac{n\omega L}{R_L}$

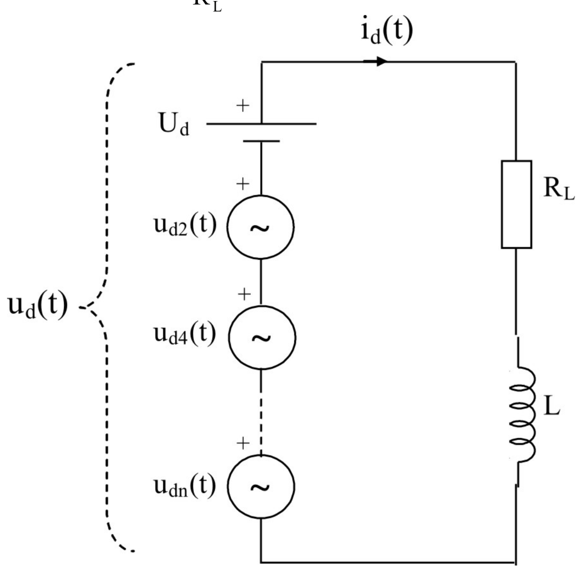


Figure 6: Equivalent voltage sources corresponding to Fourier series

Equation (3) is equal to (1.b) in case that the load current is uniform, inductance L is large enough. Equation (3) relating to the nonuniform load current, $i_d(t)$, can be partitioned as follows

$$i_d(t) = I_d + i_{df}(t) + i_{dh}(t) \quad (4)$$

Where, I_d is the DC component (average current), i_{df} and i_{dh} represent the fundamental (first) component, the harmonic components of the load current, respectively. Since $i_d(t)$ has even harmonics, $i_{df}(t)$ is equal to 0.

The source current on AC side, $i_s(t)$, is dependent on the load current, $i_d(t)$. Although the current on DC side of the converter is a rectified, accordingly unidirectional current, the current on AC side of the converter is bidirectional as shown in Fig.4. In other words, the source current has a bidirectional form of load current. In order to express the nonuniform source current, first, the DC component of load current, I_d, is converted into bidirectional form as shown in Fig.3b. This rectangular current, $i_{sa}(t)$ is expressed by Fourier series as follows,

$$i_{sa}(t) = \sum_{n=1,3,5,...}^{\infty}\frac{4}{n\pi}I_d\sin(n\omega t) \quad (5.a)$$

$$i_{sa}(t) = i_{saf}(t) + i_{sah}(t) \quad (5.b)$$

In (5.a), the current I_d is equal to (1.b). i_{saf} and i_{sah} represent the fundamental component, the harmonic components of current $i_{sa}(t)$, respectively. In order to express the nonuniform source current, second, the harmonic components of load current, $i_{dh}(t)$, are separated from (3).

$$i_{dh}(t) = \frac{2U_m}{\pi}\sum_{n=2,4,6...}^{\infty}\left(\frac{2}{n^2-1}\right)\left(\frac{-1}{\sqrt{{R_L}^2+(n\omega L)^2}}\right)\cos(n\omega t-\varphi_n) \quad (6)$$

The exact expression of nonuniform source current, $i_s(t)$ is given in (7). Where, $k=\pm 1$. The load current has a rectified form of source current. The positive portion of $i_s(t)$ is the same as $i_d(t)$ and $k=1$. On the other hand, although the negative portion of $i_s(t)$ is the inverse of $i_d(t)$, k is taken as –1.

$$i_s(t) = i_{sa}(t) + k\cdot i_{dh}(t)$$

$$i_s(t) = \sum_{n=1,3,5,...}^{\infty}\frac{8U_m}{n\pi^2 R_L}\sin(n\omega t) + k\cdot\frac{2U_m}{\pi}\sum_{n=2,4,6...}^{\infty}\left(\frac{2}{n^2-1}\right)\left(\frac{-1}{\sqrt{{R_L}^2+(n\omega L)^2}}\right)\cos(n\omega t-\varphi_n) \quad (7)$$

As seen from (7), the nonuniform source current contains both odd and even harmonics. Now, Let's deal with the input voltage of converter, $u_a(t)$, in Fig.(1). Since this voltage is dependent on the source current, $i_s(t)$, thereby the load current, it is modeled by a current controlled voltage source, which is controlled with the source current, in Fig.7. Since the load/source current is not uniform, the harmonic voltage source has a nonuniform wave form.

$$u_a(t) = u_s(t) - R_s i_s(t) - L_s \frac{di_s(t)}{dt}$$

$$u_a(t) = u_s(t) - \sum_{n=1,3,5,\dots}^{\infty} \frac{8U_m}{n\pi^2 R_L} \sqrt{R_s^2 + \frac{L_s^2}{n^2\omega^2}} \sin(n\omega t + \varphi_a) + k \cdot \frac{2U_m}{\pi} \sum_{n=2,4,6\dots}^{\infty} \left(\frac{2}{n^2-1}\right)\left(\frac{1}{\sqrt{R_L^2 + (n\omega L)^2}}\right) \sqrt{R_s^2 + \frac{L_s^2}{n^2\omega^2}} \sin(n\omega t - \varphi_n - \varphi_a) \quad (8)$$

Now, to derive the circuit relating to nonuniform structure containing harmonic currents on AC side and harmonic voltages on DC side of rectifiers, we discuss an equivalent circuit in Fig.8. This model, primarily, is a combination of Fig.5 and Fig.7.

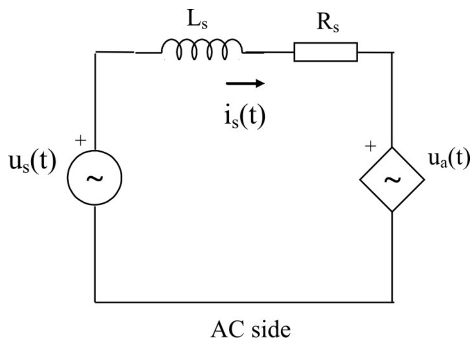


Figure 7: Representation of dependent harmonic voltage source

Since the equivalent circuit shown in Fig.8 contains both AC side and DC side of the rectifier circuit, it meets the need for exact analysis of the rectifier circuit. Owing to this approach, the switching circuit is converted into a linear circuit with regards to harmonics. Thus, any circuit analysis technique can be used to analyze the rectifier circuit.

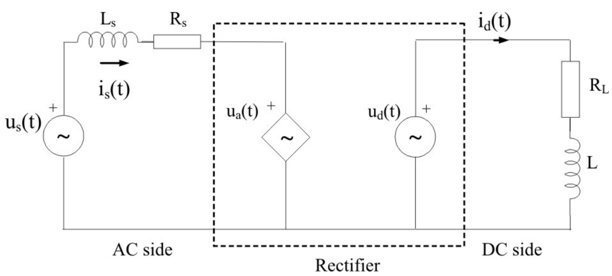


Figure 8: Exact harmonic model of rectifier circuit

From the point of harmonic studies, the equivalent circuit can be divided into two subcircuits: one is an equivalent circuit to the fundamental component and the other is an equivalent circuit to harmonics.

First, the equivalent circuit to the fundamental components on both AC side and DC side of the rectifier is discussed. In this case, the first harmonic components of dependent voltage source, $u_a(t)$, and voltage source $u_d(t)$ in the model are considered: $u_{a1}(t)$, $u_{d1}(t)$. The source voltage, $u_s(t)$ remains unchanged. According to (2), Since $u_d(t)$ has even harmonics, $u_{d1}(t)$ is equal to 0. The equivalent circuit to the fundamental components is obtained as in Fig.9.

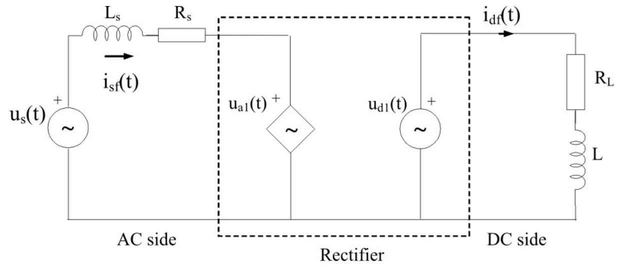


Figure 9: Equivalent circuit to fundamental components

Next, the equivalent circuit to harmonic components on both AC side and DC side of the rectifier is discussed. This equivalent circuit can be obtained under the condition that the source voltage $u_s(t)$=0 and the first component of harmonic voltage source $u_{a1}(t)$=0 in Fig.8. The equivalent circuit to harmonics is shown in Fig.10. $u_{ah}(t)$ and $u_{dh}(t)$ represent the sources relating to harmonic components. By this linear circuit, it can be obtained the desired harmonic components of rectifier as seen in Section IV.

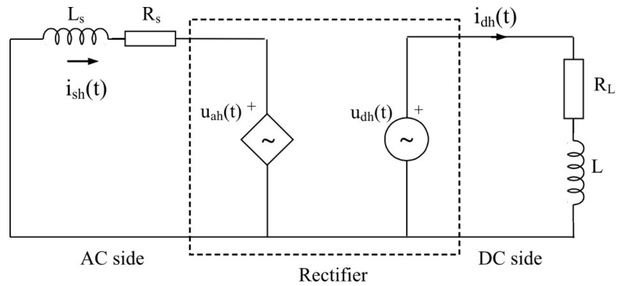
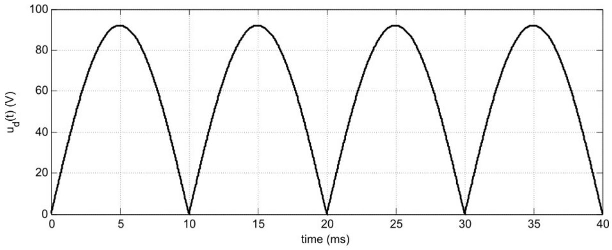


Figure 10: Equivalent circuit to harmonic components

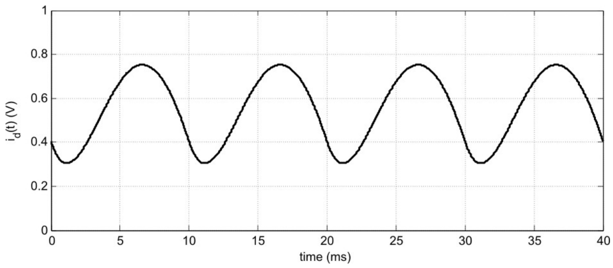
4 Simulation and experimental results

The full wave diode rectifer in Fig.1 is practised experimentally. The internal impedance, R_s and L_s, on AC side of the rectifier are 2Ω, 1mH and the load resistance R_L and load inductance L on DC side are 96Ω and 200mH, the max. value of the source voltage U_m is 92V and the frequency is 50Hz. The diodes are assumed to be ideal. The measurements are obtained by a power quaility analyzer.

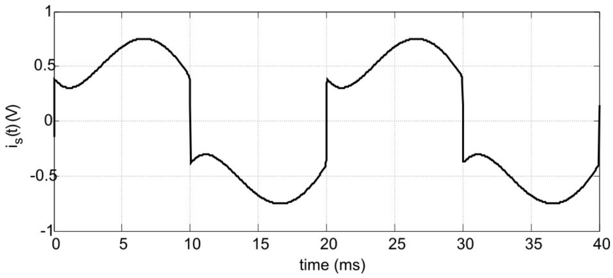
The simulations results and experimental results are given in Fig.11 and Fig.12, respectively. The load voltage, $u_d(t)$, load current, $i_d(t)$ and source current, $i_s(t)$ waveforms relating to the exact equivalent circuit in Fig.8 are given in Fig.11.



(a) Load voltage, $u_d(t)$



(b) Load current, $i_d(t)$

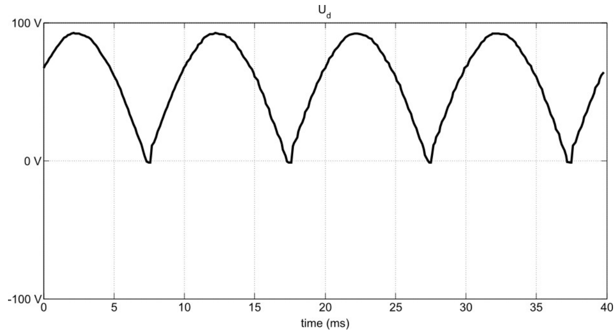


(c) Source current , $i_s(t)$

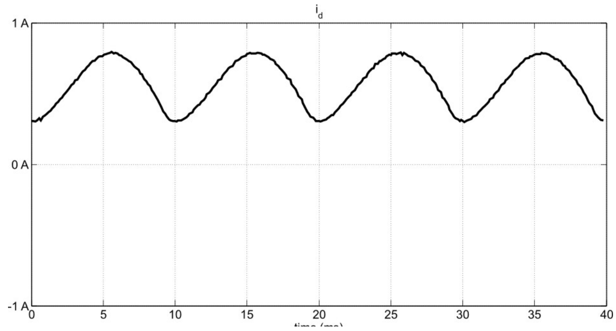
Figure 11: Simulation results relating to the equivalent circuit

The experimental results to load voltage, $u_d(t)$, load current, $i_d(t)$, and source current, $i_s(t)$ are given in Fig.12.

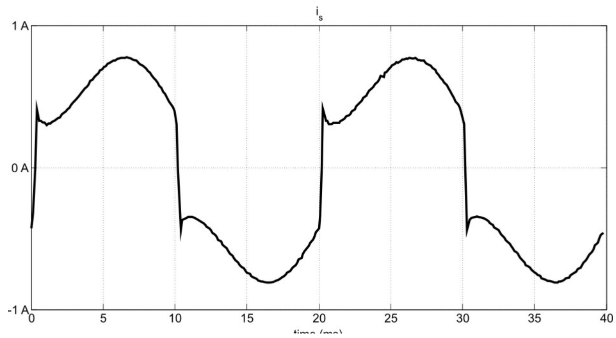
The fundamental component of source current, $i_{sf}(t)$, relating to Fig.9 are given in Fig.13. The desired harmonic components can be easily obtained by the proposed model. The waveform of harmonic components, $i_{sh}(t)$, except the fundamental component, of source current, relating to Fig.10 is given in Fig.14.



(a) Load voltage, $u_d(t)$



(b) Load current, $i_d(t)$



(c) Source current , $i_s(t)$

Figure 12: Experimental results relating to Fig.1

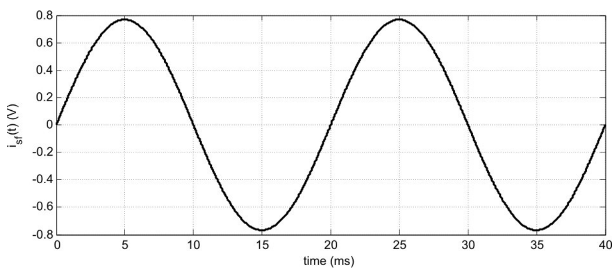


Figure 13: Simulation result relating to fundamental component, $i_{sf}(t)$

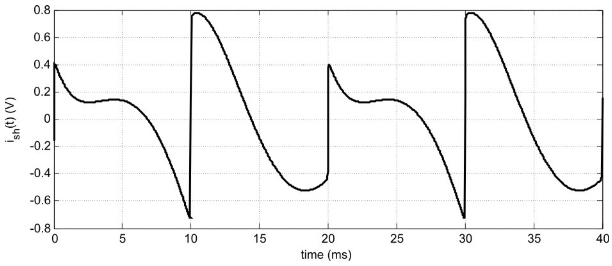


Figure 14: Simulation result relating to harmonic component, $i_{sh}(t)$

5 Conclusion

For an accurate harmonic analysis of converter circuits, the adequate models of harmonic sources are required. For a general analysis, the paper describes the exact equivalent circuit containing nonuniform current harmonics on AC side and voltage harmonics on DC side of rectifiers. The exact formulation relating to the solution and the model is also given. The main contribution of paper is that the circuit model is general because it contains both source/load impedances and nonuniform load currents. By the proposed model, the switching circuit is converted into a linear and non-switched circuit with regards to harmonics. The desired harmonic components are easily obtained by the proposed model. This appraoach can be also applied to other rectifier structures.

References

1. H. W. Dommel, A. Yan, and S. Wei, "Harmonics from transformer saturation", *IEEE Trans. on Power Delivery*, vol. 1. no. 2. pp. 209-215, April 1986.
2. A. Gawad and A.F. Zagazig, "Studying the impact of different lighting loads on both harmonics and power factor", Universities Power Engineering Conference, UPEC 2007.
3. Z.M. Lian, W.X. Bo and Y.X. Lang, "A novel smart power management integrated circuit used for fluorescent lamp", Power Electronics and Motion Control Conf., pp.1568–1572, vol.3, 2004.
4. J.A. Fuentes, et al., "Harmonic model of electronically controlled loads", IEEE Power Engineering Society Summer Meeting, Vol. 3, pp. 1805 – 1810, 2000.
5. W. Wichakool, A.T. Avestruz, R.W. Cox, S.B. Leeb, "Modeling and estimating current harmonics of variable electronic loads", *IEEE Trans. on Power Electronics*, Vol.24, no.12, pp.2803-2811, Dec.2009.
6. X.P. Zhang, "Frequency-dependent simple harmonic model of synchronous machines", Power Engineering Review, IEEE, May 2000.
7. IEEE PES Harmonic Working Group, "Characteristics and modeling of harmonic Sources – Power electronic devices" *IEEE Trans. on Power Delivery,* vol. 4. no. 1, pp 791-800. Oct. 2001.
8. T. Tanaka and H. Akagi, "Proposal for a new equivalent circuit of diode rectifiers with respect to harmonics", Power Conversion Conference, pp 508 - 513, Yokohama 1993.
9. N.K. Medora and A. Kusko, "Computer-aided design and analysis of power-harmonic filters", *IEEE Trans. on Industry Applications,* Vol. 36. no. 2, pp. 604-613. Mar/Apr 2000.
10. G. W. Chang and Y. Liu, "Modeling of nonlinear components for harmonic studies", IEEE Power Engineering Society General Meeting, Vol. 2, Taiwan, 2003.
11. R. Ghandehari, S. Mohamadian and A. Shoulaie, "A new approach to AC/DC converters modelling in time domain for harmonic analysis", Power Quality Conf. pp.1-5, Iran, 2010.
12. K.L Lian, B.K. Perkins, P.W. Lehn, "Harmonic analysis of a three-phase diode bridge rectifier based on sampled-data model", *IEEE Trans. on Power Delivery*, Vol.23, no.2, pp.1088-1096, Apr. 2008.
13. Z. Zheyu, Z. Xudong. W. Zhenxing, Z. Yunping, "Analysis on harmonic current of three-phase bridge uncontrolled rectifier", Asia-Pacific Power and Energy Engineering Conference (APPEEC), pp. 1-4, China, 2010.
14. D. Maheswaran, N. Rajasekar, "Modeling and simulation of 6-pulse rectifier drives with impacts to input current harmonics", Int. Conf. on Computing, Electronics and Electrical Technologies (ICCEET), pp.477-478, India, 2012.
15. A. Hernadi, M.A. Taufik, "Modeling and simulation of 6-pulse and 12-pulse rectifiers under balanced and unbalanced conditions with impacts to input current harmonics", 2nd Asia Int. Conf. on Modeling & Simulation, pp.1034-1038, 2008.
16. H. Pham, H. Jung, T. Hu, "State-space approach to modeling and ripple reduction in AC-DC converters", *IEEE Trans. Control Systems Tech.*, vol.1, no.99, pp.1-7, 2012.
17. L. Hu and R. E. Morrison, "AC side equivalent circuit-based method for harmonic analysis of a converter system", *IEE Proc.Electr. Power Appl.*, vol. 146, no. 1, pp. 103–110, Jan. 1999.
18. J.M. Cano, et al., "New transfer functions for an accurate estimation of harmonic distortion in AC/DC converters working under unbalanced conditions," *IEEE Trans. Industry Applications*, vol.37, pp. 642-649, 2001.
19. M. Sakui, H. Fujita, M. Shioya, "A Method for calculating harmonic currents of a three-phase bridge uncontrolled rectifier with DC filter", *IEEE Trans.*

Industrial Electronics, vol. 36, no.3, pp.434-440, Aug.1989.
20. M. Bauta, M. Grotzbach, "Noncharacteristic line harmonics of AC/DC converters with high DC current ripple"", *IEEE Trans. on Power Delivery*, Vol.15, no.3, pp.1060-1066, Jul. 2000.
21. W. Wei, M. Xiaojun. W. Shuguang, H. Zhaozhao, "Harmonic amplitudes calculation method and simulation of diode-bridge rectifier's output current", Int. Conference on Measuring Technology and Mechatronics Automation (ICMTMA), pp.382-385, China, 2010.
22. V.A Katic, M. Knezevic, and D. Graovac, "Application-oriented comparison of the methods for AC/DC converter harmonics analysis", *IEEE Trans. Industrial Electronics,* vol. 50, pp.1100-1108, 2001.
23. Y. Sun, et al., "A harmonically coupled admittance matrix model for AC/DC converters", *IEEE Trans. Power Systems,* vo1.22, pp.1574-1582, 2007.
24. R.W. Erickson and D. Maksimovic, "Fundamentals of Power Electronics", Kluwer Academic Publishers, 2nd Ed., NY, 2004.

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