Informacije MIDEM Journal of Microelectronics, Electronic Components and Materials Vol. 45, No. 1 (2015), 29 – 38

# The parameter estimation of the electrothermal model of inductors

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**Abstract:** This paper presents the electrothermal model of inductors dedicated to the analysis of dc-dc converters in SPICE and the proposed method of determining parameters of this model. The parameter estimation algorithm of this model is described in detail. The results of verification of the correctness of the model and the estimation procedure for arbitrarily selected choking - coils are presented. Very good agreement between the calculated and measured characteristics of the considered choking-coils was obtained.

Keywords: Inductors; modelling; parameters estimation; self-heating

### Ocena parametrov elektrotermičnega modela tuljav

**Izvleček:** Članek opisuje elektrotermični model in določevanje parametrov tuljav, ki se uporabljajo v dc-dc konverterjih v SPICE. Natančno je opisan algoritem določevanja parametrov modela. Predstavljeni so rezultati verifikacije modela in postopek ocenitve parametrov na izbranih tuljavah. Rezultati simulacij se dobro ujemajo z meritvami.

Ključne besede: Tuljava; modeliranje; ocean parametrov; samogretje

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#### 1 Introduction

Inductors are important components of switchedmode power converters [1 - 4]. Properties of such converters depend on the properties of their structural components, i.e. the ferromagnetic core and the winding. Ferromagnetic materials used to build the core of the inductor are characterized by magnetization hysteresis characteristics. The magnetic permeability of the core, which is proportional to the inductance of inductors is a non-linear function of magnetic force and temperature [5 - 11].

In designing electronic circuits the computer programs dedicated to their analysis are used. Currently, one of most popular programs for this analysis is SPICE software [12 – 15]. The credibility of calculation results depends on the accuracy of the models of the used elements [16]. The inductor models typically use a linear model of the coil or non-linear model of the core and the linear model of the winding [2, 13, 17]. Nonlinear models of the core were presented in [3, 10, 13, 18, 19], but various modifications of the Jiles–Atherton model are the most commonly used models [6, 7, 11, 18, 19,

20]. This model does not take into account such an important phenomenon as self-heating.

In papers [3, 18] the electrothermal model of the chocking-coil for SPICE using the electrothermal core model presented in [11] is proposed. The electrothermal model of the choking-coil is devoted to calculate parameters of its model for the inductor used in the analyzed circuit. Therefore, it is important to prepare algorithm parameter estimation of such a model. This paper presents a modified form of the electrothermal model of the inductor, proposes the method for determining parameters of the model and provides an example of the results of calculations and measurements to illustrate the correctness of the elaborated method.

## 2 The electrothermal model of the inductor

The presented electrothermal model of the choking – coil takes into account electrical phenomena occur-

ring in the winding, magnetic phenomena occurring in the core and thermal phenomena in the core and the winding. Due to the fact that the choking – coil core is made of soft magnetic material the hysteresis of the magnetization curve can be omitted in the model [6]. The considered electrothermal model of the choking – coil has the form of a sub-circuit of SPICE. The network representation of the elaborated model is presented in Figure 1. The model is composed of three blocks. The first block is the main circuit and it includes a series connection of controlled voltage sources  $E_{LS'} = E_{RS'}$ the voltage source  $V_L$  with the zero value and the coil with inductance L equal to 2  $\mu$  H and the parallelly connected capacitor  $C_w$  modeling interturn capacitance of the winding.



**Figure 1:** Network representation of the electrothermal model of the inductor

The voltage source  $V_L$  monitors the value of the current of the choking – coil. The coil L make it possible to calculate the time derivative of the current of the choking – coil. E<sub>LS</sub> represents the voltage drop on non-linear inductance of the choking – coil and is described by the formula [3]

$$E_{LS} = w_S \cdot \frac{f \cdot V_L}{(f+f_b) \cdot L} \cdot L_S = w_S \cdot \frac{f \cdot V_L}{(f+f_b) \cdot L} \cdot \frac{z^2 \cdot S_{Fe} \cdot B_{sat} \cdot A}{I_{Fe} \cdot (|H| + A)^2 + A \cdot B_{sat} \cdot I_p / \mu_0}$$
(1)

where z denotes the number of turns in the choking– coil winding,  $V_L$  - voltage on the coil L,  $S_{Fe}$  - effective cross-section area of the core,  $B_{sat}$  – saturation magnetic flux density, *H* - magnetic force in the core,  $I_{Fe}$  - magnetic path in the core, *A* – the field parameter,  $I_{p}$  - air gap length in the core,  $\mu_{o}$  – permeability of free air, which amounts to 12.57 • 10<sup>-7</sup> H/m, *dB/dH* – magnetic permeability of the core, *f* – the frequency of the inductor current,  $f_{b}$  - reference frequency.

The resistor  $R_{so}$  represents series resistance of the inductor at temperature  $T_o$ . The value of this resistance is described by formula:

$$R_{SO} = \rho \cdot \frac{l_d}{S_d} \tag{2}$$

where  $\rho$  is resistivity of copper equal to 1.72 • 10<sup>8</sup>  $\Omega$ •m at temperature 20° C,  $I_d$  is the length of winding, and  $S_d$  is the cross-section of the coil wire.

In turn, the controlled voltage source  $\mathrm{E}_{_{\mathrm{RS}}}$  is described by:

$$E_{RS} = V_{RS} \cdot \alpha_{\rho} \cdot (T_{U} - T_{0}) + \frac{l}{d} \cdot \sqrt{\mu_{0} \cdot \rho \cdot (\mathbf{l} + \alpha_{\rho} \cdot (T_{U} - T_{0}))} \cdot 2 \cdot (I - I_{av}) \cdot \sum_{n=1}^{4} \sqrt{n \cdot f} \cdot \left(a_{n} \cdot \cos\left(\frac{2 \cdot \pi \cdot f}{t}\right) + b_{n} \cdot \sin\left(\frac{2 \cdot \pi \cdot f}{t}\right)\right) + P_{R} \cdot \frac{I}{I_{sk}^{2}}$$
(3)

In the equation (3) there are three components. The first one models the dependence of series resistance on temperature. The  $V_{RS}$  is a component of the voltage across the resistor  $R_{SO'} \alpha_p$  is the temperature coefficient of resistivity of copper, which amounts to  $4.45 \cdot 10^{-3} \text{ K}^{-1}$  and  $T_u$  is temperature of the winding. The second component models the additional voltage drop at the choking–coil which is a result of the skin effect. To describe these phenomena one takes into account the fact that the current of the choking–coil operating in the dc-dc converter has a periodic triangular waveform. This waveform is modeled with a Fourier series, wherein the number of components is limited to four. The Fourier series coefficients of the model are described by:

$$a_{n} = \frac{2 \cdot \cos\left(2 \cdot n \cdot \pi \cdot (d_{1} - 0.5) - (-1)^{n}\right)}{d_{1} \cdot n^{2} \cdot \pi^{2} \cdot (1 - d_{1})}$$
(4)

$$b_n = \frac{\sin(2 \cdot n \cdot \pi \cdot (d_1 - 0, 5) - (-1)^n}{2 \cdot \pi^2 \cdot n^2 \cdot d_1 \cdot (1 - d_1)}$$
(5)

where *I* is choking–coil current, *d* – diameter of the coil wire,  $I_{av}$  – average value of the coil current calculated in the auxiliary block,  $d_i$  - duty of the converter control signal, and *f* – frequency of this signal. The third component in the formula (3) represents the choking–coil voltage drop resulting from energy losses in the core. The  $P_R$  component describe energy losses in the core,  $I_{sk}$  is the RMS value of the choking–coil current.

In the auxiliary block the following are determined: magnetic force *H*, magnetic flux density *B*, the time derivative of the magnetic flux density *DB*, field parameter *A*, maximum and average values of the magnetic flux density and of the current, coefficient *c* defining the influence of the Curie temperature  $T_c$  on the value of the magnetic flux density. Inductance of the inductor is proportional to the magnetic permeability of the core corresponding to the characteristics *B*(*H*) slope [2, 6, 13]. To determine the value of the magnetic flux density the formula described in [6, 21] is used:

$$B = B_{sat} \cdot \frac{H}{|H| + A} \tag{6}$$

where  $B_{sat}$  is the saturation flux density of the core.

On the other hand, the value of the magnetic force is calculated by the formula [5]:

$$H = \frac{z \cdot I - \frac{B \cdot l_p}{\mu_0}}{l_{Fe} + l_p} \tag{7}$$

In the auxiliary block, the field parameter *A*, which makes it possible to take into account the influence of temperature on the magnetization curve and inductance of the inductor, is also determined. The dependence of the parameter *A* on temperature is described by the empirical formula:

$$A = A_0 \cdot \exp\left[\left(-T_R + T_a\right)/\alpha_T\right] \tag{8}$$

where  $\alpha_{T}$  is the temperature coefficient of the parameter *A*.

It should be noted that the saturation flux density in the core also strongly depends on temperature and the inclusion of this impact has been expressed by the dependence [6, 10, 11]:

$$B_{sat} = B_{sat0} \cdot \left[1 + \alpha_{BS} \cdot \left(T_R - T_0\right)\right] \cdot c \tag{9}$$

where  $B_{sat0}$  is the saturation flux density at temperature  $T_0$  and  $a_{BS}$  – the temperature coefficient of  $B_{sat}$ .

The *c* coefficient was defined by:

$$c = \begin{cases} 1 & \text{for } T_R < T_C \\ 1 - 0.1 \cdot (T_R - T_C) & \text{for } T_R < T_C + 10K \\ 0 & \text{for } T_R > T_C + 10K \end{cases}$$
(10)

where  $T_{R}$  denotes temperature of the core.

On the other hand, to calculate the average and peakto-peak values of the current, and the magnetic flux density, two detectors are defined: the peak-to-peak value detector and the average value detector, consisting of the two-terminal networks  $R_1C_1$ ,  $R_2C_2$  and  $R_{11}C_{11}$ ,  $R_{21}C_{21}$ , diodes  $D_1$  and  $D_{11}$ , the controlled voltage sources  $E_1$  and  $E_{11}$ , respectively representing the inductor current and the magnetic flux density of the core.

The thermal model is used to determine the core temperature  $T_{\mu}$  and the winding temperature  $T_{\mu}$  of the inductor using the compact model proposed in [6, 12, 16, 22, 23]. This model includes two controlled current sources, representing power losses in the core G<sub>PR</sub> and in the winding  $\mathsf{G}_{_{\mathsf{PU'}}}$  respectively. The included in this two-terminal circuits  $R_{_{thR'}}$   $C_{_{thR}}$  and  $R_{_{thU'}}$   $C_{_{thU}}$  represent thermal time constants of the core and the winding, so that it is possible to take into account the phenomena of self-heating. These time constants fulfill equations describing the relation between the controlled current sources and G<sub>PU1</sub> used for modeling the thermal coupling between the core and winding. The currents of these sources are respectively 0.8 G<sub>PR</sub> and 0.8 G<sub>PU</sub>. Depending how one defines a power loss in the winding, G<sub>PU</sub> includes resistive losses and the skin effect . The losses in the winding are described by the formula:

$$P_{U} = \rho \cdot I^{2} \cdot \left[1 + \alpha_{\rho} \cdot (T_{U} - T_{0})\right] + l/d \cdot \sqrt{\mu_{0} \cdot \rho \cdot f \cdot (1 + \alpha_{\rho} \cdot (T_{U} - T_{0}))} \cdot 2 \cdot \frac{4}{\sqrt{n \cdot f}} \left(\alpha_{n} \cdot \cos\left(\frac{2 \cdot \pi \cdot f}{t}\right) + b_{n} \cdot \sin\left(\frac{2 \cdot \pi \cdot f}{t}\right)\right) \cdot \left(I_{mx} - I_{av}\right)^{2}$$
(11)

where  $I_{mx}$  is the maximum coil current calculated in the auxiliary block.

In turn, the core losses are described by [10]:

$$P_{R} = V_{e} \cdot \left(\frac{DB}{2}\right)^{\beta - \alpha} \cdot \left(1 + D \cdot \left(T_{R} - T_{m}\right)\right)^{2} \cdot \frac{P_{V0}}{T} \cdot \int_{0}^{T} \left|\frac{dB}{dt}\right|^{\alpha} dt \qquad (12)$$

where  $V_e$  denotes the equivalent volume of the core,  $P_{vo}$  are volumial power losses in the core, *DB* is the magnitude of flux density, *D* – the square temperature coefficient of power losses  $P_{vo'}$ , *T* - period of a inductor current,  $\alpha$  and  $\beta$  are exponents in the dependence of core losses based on frequency and amplitude of the flux density in the choking–coil, respectively,  $T_m$  is the temperature, at which losses are minimal.

#### 3 Parameter estimation

The presented model is described by 20 parameters that can be divided into 3 groups:

- a. electrical parameters,
- b. magnetic parameters,
- c. thermal parameters.

The proposed estimation algorithm uses the concept of local estimation described in [22, 24]. According to this concept, the model parameters are estimated in groups on the basis of the measured characteristics of the inductor operating in specific conditions.

The magnetic parameters of the choking–coil corresponding to the ferromagnetic core reactor can be divided into three groups:

- The parameters of ferromagnetic material, of which the core is made, related to the hysteresis loop, such as the saturation flux density  $B_{sato}$ , the Curie temperature  $T_{cr}$  the field parameter A, the air gap length  $I_{p}$ , the temperature coefficient of saturation flux density changes  $a_{BS}$ , the temperature coefficient  $a_{\tau}$  of the magnetic field parameter,
- The geometric parameters of the core, such as the magnetic path length in the core  $I_{Fe'}$  the equivalent value of the core volume  $V_{e'}$  the effective cross-section area of the core  $S_{Fe'}$
- The ferromagnetic material parameters corresponding to core losses such as  $P_{\nu\rho'}D$ ,  $\alpha$ ,  $\beta$ .

Some parameters associated with the magnetic material used to construct the ferromagnetic core can be read directly from the catalog data supplied by manufacturers e.g. the saturation flux density  $B_{sat0}$  and the Curie temperature  $T_c$  [22].

In order to determine the temperature coefficient of saturation flux density changes the designer needs to:

- 1. Read from the catalog characteristics, eg, [25, 26], the value of the saturation flux density  $B_{sat0}$  at the reference temperature  $T_0$  and the value of this parameter  $B_{sat1}$  at a different temperature  $T_1$ .
- 2. Calculate the value of the temperature coefficient of saturation flux density changes according to the formula [22] :

$$\alpha_{BS} = \frac{B_{sat1} / B_{sat0} - 1}{T_1 - T_0}$$
(13)

The geometric parameters of the cores should be read from the catalog data or should be determined basis of the dimensions of the core and calculated using the basic geometrical relationships. For example, to determine the geometrical parameters of the ring core one should:

1. determine the dimensions of the core (Fig. 2), i.e. the outer diameter  $d_z$ , the inner diameter  $d_w$  and height  $h_R$  (these data are usually contained in the name of the core, e.g. RTP 26,9 x14, 5x11)



Figure 2: Dimensions of the ring core

2. calculate the magnetic path length in the core  $I_{re}$  using the formula

$$l_{Fe} = \pi/2 \cdot \left(d_z + d_w\right) \tag{14}$$

3. calculate the effective cross-section area of the core  $S_{F_e}$  using the formula:

$$S_{Fe} = \frac{\left(d_z - d_w\right) \cdot h_R}{2} \tag{15}$$

 calculate the equivalent value of the core volume V<sub>p</sub> by:

$$V_{e} = \frac{\pi \cdot \left(d_{z}^{2} - d_{w}^{2}\right) \cdot h_{R}}{4}$$
(16)

In order to determine the values of the parameters *A*, *w<sub>s</sub>*, and *I<sub>p</sub>* it is necessary to measure the dependence of inductance *L* on the DC current using the measurement system described in [27]. The measurement should be performed at the frequency  $f << f_b$ . In the measured characteristics of *L(i)*, whose typical course is shown in Figure 3 one should select 3 points: X<sub>1</sub>(*I<sub>p</sub>*, *L<sub>1</sub>*), X<sub>2</sub>(*I<sub>2</sub>*, *L<sub>2</sub>*) and X<sub>3</sub>(*I<sub>3</sub>*, *L<sub>3</sub>*). Then, the following system of equations must be solved for *w<sub>s1</sub> I<sub>p</sub>* and *A*:



**Figure 3:** Typical course of the dependence of inductance of the inductor on the dc part of its current

$$\begin{bmatrix}
L_{1} = \frac{w_{s} \cdot z^{2} \cdot S_{Fe} \cdot B_{sat} \cdot A}{l_{Fe} \cdot \left(\frac{z \cdot I_{1} - B_{sat} - A \cdot x + \sqrt{B_{sat}^{2} + z^{2} \cdot I_{1}^{2} + A^{2} \cdot x^{2} + 2 \cdot A \cdot x \cdot (B_{sat} + z \cdot I_{1}) - 2 \cdot B_{sat} \cdot z \cdot I_{1}} + A\right)^{2} + A \cdot B_{sat} \cdot l_{p} / \mu_{0} \\
\end{bmatrix}$$

$$\begin{bmatrix}
L_{2} = \frac{w_{s} \cdot z^{2} \cdot S_{Fe} \cdot B_{sat} \cdot A}{l_{Fe} \cdot \left(\frac{z \cdot I_{2} - B_{sat} - A \cdot x + \sqrt{B_{sat}^{2} + z^{2} \cdot I_{2}^{2} + A^{2} \cdot x^{2} + 2 \cdot A \cdot x \cdot (B_{sat} + z \cdot I_{2}) - 2 \cdot B_{sat} \cdot z \cdot I_{2}} + A\right)^{2} + A \cdot B_{sat} \cdot l_{p} / \mu_{0} \\
\end{bmatrix}$$

$$\begin{bmatrix}
L_{3} = \frac{w_{s} \cdot z^{2} \cdot S_{Fe} \cdot B_{sat} \cdot A}{l_{Fe} \cdot \left(\frac{z \cdot I_{3} - B_{sat} - A \cdot x + \sqrt{B_{sat}^{2} + z^{2} \cdot I_{3}^{2} + A^{2} \cdot x^{2} + 2 \cdot A \cdot x \cdot (B_{sat} + z \cdot I_{3}) - 2 \cdot B_{sat} \cdot z \cdot I_{3}} + A\right)^{2} + A \cdot B_{sat} \cdot l_{p} / \mu_{0}$$

$$\begin{bmatrix}
L_{3} = \frac{w_{s} \cdot z^{2} \cdot S_{Fe} \cdot B_{sat} \cdot A}{l_{Fe} \cdot \left(\frac{z \cdot I_{3} - B_{sat} - A \cdot x + \sqrt{B_{sat}^{2} + z^{2} \cdot I_{3}^{2} + A^{2} \cdot x^{2} + 2 \cdot A \cdot x \cdot (B_{sat} + z \cdot I_{3}) - 2 \cdot B_{sat} \cdot z \cdot I_{3}} + A\right)^{2} + A \cdot B_{sat} \cdot l_{p} / \mu_{0}$$

$$\begin{bmatrix}
L_{3} = \frac{w_{s} \cdot z^{2} \cdot S_{Fe} \cdot B_{sat} \cdot A}{l_{Fe} \cdot \left(\frac{z \cdot I_{3} - B_{sat} - A \cdot x + \sqrt{B_{sat}^{2} + z^{2} \cdot I_{3}^{2} + A^{2} \cdot x^{2} + 2 \cdot A \cdot x \cdot (B_{sat} + z \cdot I_{3}) - 2 \cdot B_{sat} \cdot z \cdot I_{3}} + A}\right]^{2} + A \cdot B_{sat} \cdot l_{p} / \mu_{0}$$

Where  $x = \mu_0 \cdot l_p \cdot (l_{Fe} + l_p)$ .

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In order to determine the value of the temperature coefficient  $\alpha_{\tau}$ , it is necessary to measure the dependence of *L* (*i*) for the temperature  $T_{\tau} > T_{o\tau}$  and then to determine the value of the parameter  $A_{\tau}$  at temperature  $T_{\tau}$ using the formula (17). The value of  $\alpha_{\tau}$  is given by:

$$\alpha_T = \frac{T_0 - T_R}{\ln(A_1/A_0)}$$
(18)

In order to determine the parameters describing losses in the core:

1. one should read from the catalog characteristics of the core material describing the dependence of power losses density  $P_v$  on the amplitude of the magnetic flux density  $(B_m)$  at constants frequency f, the coordinates of two points  $X_4(B_m, P_{V1})$  and  $X_5$  $(B_{m2'}, P_{V2})$ . The typical course of such characteristics is shown in Figure 4. Next, one should calculate the value of the  $\beta$  coefficient by the formula:

$$\beta = \frac{\log(P_{v1} / P_{v2})}{\log(B_{m1} / B_{m2})}$$
(19)

2. to determine the coefficient  $\alpha$  one should read from the catalog characteristics describing the dependence of the power density of the core loss on frequency (Fig. 4b) at the known amplitude  $B_m$ in points  $X_6(f_{\gamma}, P_{\gamma 3})$  and  $X_7(f_2, P_{\gamma 4})$  and calculate the value of the factor  $\alpha$  from the formula:

$$\alpha = \frac{\log(P_{v_3} / P_{v_4})}{\log(f_1 / f_2)}$$
(20)

3. to determine the parameter  $P_{v_0}$  it is necessary to read point e.g.  $X_6(f_{\gamma}, P_{\gamma_3})$  from the characteristics describing the dependence of power density losses  $P_v$  on frequency at the constant value of  $B_m$  (Fig. 4b) and next, calculate the value of  $P_{v_0}$  from:

$$P_{vo} = \frac{P_{v3}}{f_1^{\alpha} \cdot B_m^{\beta} \cdot (2 \cdot \pi)^{\alpha} \cdot [0.6336 - 0.1892 \cdot \ln(\alpha)]}$$
(21)

4. in the catalog characteristics  $P_v(T)$  at first one should read the value of temperature  $T_m$  at which the characteristics of  $P_v(T)$  reaches the minimum at point  $X_8(T_{m'}, P_{VS})$  (Fig. 4c), then one should select point  $X_9(T_{e'}, P_{Ve})$  of some characteristics and calculate the parameter *D* by the formula:

$$D = \frac{Pv_6 - Pv_5}{Pv_5 \cdot (T_6 - T_m)^2}$$
(22)

In turn, the value of the parameter  $f_b$  associated with the dependence defining the output voltage of the controlled voltage source  $E_{LS}$  can be determined from the dependence describing the characteristics of the magnetic permeability  $\mu$  of the core of the frequency  $\mu(f)$ , whose typical course is shown in Figure 5.

The frequency  $f_{h}$  is calculated by the formula

$$f_b = \frac{f_1 \cdot \mu_1 - f_2 \cdot \mu_2}{\mu_2 - \mu_1}$$
(23)

where in the calculations the coordinates of two points  $X_{10}$  ( $f_{\gamma}$ ,  $\mu_{\gamma}$ ) and  $X_{11}$  ( $f_{\gamma}$ ,  $\mu_{2}$ ) lying on the curve  $\mu(f)$  were used.

To the electrical parameters appearing in the description of the electrothermal model of the inductor belong



Figure 4: Dependences of power losses density on the amplitude of the flux density (a), frequency (b) and temperature (c)



**Figure 5:** Typical dependence of magnetic permeability of the ferromagnetic core on frequency

also length of the winding  $I_d$  and cross-section of the coil wire  $S_d$ . These parameters are used to determine series resistance of the coil. The length of the winding for the ring core is estimated by calculating the product of the number of turns z and the girth of cross-section of the core, assuming that it is rectangular, by the formula:

$$l_{d} = 2 \cdot z \cdot (h_{R} + ((d_{z} - d_{w})/2)))$$
(24)

In turn, the cross-sectional area of the wire is calculated on the basis of simple geometric formulas and the known wire diameter  $d_3$ .

The capacitor  $C_{w}$  is determined by the formula:

$$C_{w} = \left(f_{r}^{2} \cdot 4 \cdot \pi^{2} \cdot L_{0}\right)^{-1}$$
<sup>(25)</sup>

where  $f_r$  is resonant frequency of the inductor,  $L_o$  is the inductance value for  $I_{DC} = 0$ . The resonant frequency of the chocking - coil can be read from the course of the dependence of the impedance module, whose typical course is shown in Figure 6, on frequency.



**Figure 6:** Typical dependence of the module of inductor impedance on frequency

To determine the thermal parameters  $a_{i,} \tau_{tht'} R_{th'}$  it is necessary to perform measurements of their own transient thermal impedance of the winding  $Z_{thu}(t)$  and of the core  $Z_{thr}(t)$ , as well as the mutual transient thermal impedance between the core and the winding  $Z_{thur}(t)$ using the method described in [28]. Based on the measured waveforms  $Z_{thu}(t)$ ,  $Z_{thr}(t)$  and  $Z_{thur}(t)$  the values of capacitance and thermal resistance are calculated using the method described in [22, 23].

#### 4 Experimental results

In order to verify the correctness of the proposed method of estimating parameters of the inductor, the values of the parameters of two arbitrarily selected inductors with ferromagnetic cores were estimated and the calculated and measured characteristics of these inductors were compared. The investigations were performed for two inductors containing ring cores of the same size (26.9 mm x 14.5 mm x 11 mm). The first one was the core RTF of ferrite material F-867 and the other

was the core RTP of powdered iron from the material T106 -26. On both the cores 20 turns of the enameled copper wire of 0.8 mm diameter were wound. Using the estimation algorithm proposed in the previous section, the values of all the model parameter values were read or calculated and collected in Table 1.

The measured and calculated characteristics of the considered inductors are shown in Figures 7 – 8. In these figures the results of measurements are denoted as points, whereas the results of electrothermal analysis are represented by lines.

**Table 1:** Values of parameters of the electrothermal model of inductors with the cores RTP T106-26 and RTF F 867.

Parameter	B <sub>sat0</sub> [T]	l <sub>p</sub> [μm]	T <sub>c</sub> [K]	A [A/m]	α <sub>вs</sub> [1/K]
RTP T106-26	1.38	14	1023	4024	2.8.10-3
RTF F867	0.5	0.1	488	260	2.8.10-3
Parameter	ws	I <sub>Fe</sub> [mm]	V <sub>e</sub> [m <sup>3</sup> ]	S <sub>Fe</sub> [m <sup>2</sup> ]	z
RTP T106-26	0.5	64.99	4.43·10 <sup>-6</sup>	68.2·10 <sup>-6</sup>	20
RTF F867	0.5	62.8	3.14.10-6	50·10 <sup>-6</sup>	20
			$P_{v0}$		
Parameter	S <sub>d</sub> [m <sup>2</sup> ]	l₄ [m]	[kW/m³]	D [K <sup>-2</sup> ]	α
RTP T106-26	502·10 <sup>-9</sup>	0.6	2	0	1.59
RTF F867	502·10 <sup>-9</sup>	0.6	100	0.5·10 <sup>-6</sup>	1.02
Parameter	β	α <sub>т</sub> [1/K]	T <sub>m</sub> [K]	f <sub>b</sub> [kHz]	d[mm]
RTP T106-26	2.15	100.10 <sup>3</sup>	368	546	0.8

Figure 7 shows the dependence of inductance on the DC current of the inductor containing the powder core RTP T106 -26 (Fig. 7a) and the inductor with the ferrite core RTF F867 (Fig.7b) The tests were performed at frequency of 100 kHz for two ambient temperatures equal to 23 and 75°C. As you can see, good agreement between the results of measurements and calculations was obtained. For both the considered choking-coil the dependence L(i) is a decreasing function of the current, where the choking-coil with the ferrite core with the same geometrical dimensions achieved a higher value of inductance, moreover a wider range of changes in its value was observed. A decrease in inductance of the ferrite core (even two hundreds times) was much larger than for the core of the powdered iron (about 30 %). The different courses of the dependence L(i) for both the inductors were due to the non-linear magnetization curve of ferromagnetic cores. It is worth noticing that the course of the dependence L(i) for the choking-coil with the ferrite core showed the visible influence of the ambient temperature on its course, while for the inductor with the powder core such influence is not observed. In the characteristics of the choking-coil with the ferrite core an increase in tempera-





**Figure 7:** Measured and calculated dependence of inductance of inductors with the powder (a) and ferrite (b) cores on the current

Figure 8 shows the dependences of the module of impedance of the choking–coils with the considered cores on frequency at the constant values of the DC current. As it is visible, good agreement between the measurements and calculations results was obtained. By considering winding capacitance in the electrothermal model of the choking–coil the resonance on these characteristics was obtained, which corresponds to the obtained measurement results. The value of the resonant frequency for the ferrite core increases with an increase of the DC current, whereas for the powder core it oscillates in the range of 1.3 MHz to about 2.3 MHz.

In order to illustrate the influence of the nonlinearity of the inductor and the self-heating phenomena in this element on characteristics of dc-dc converters, the results of calculations (lines) and measurement (points) of the boost converter with the core RTP T106-26 [29] were presented in Figs. 9 and 10. In Fig.9 calculated and measured dependences of the output voltage V<sub>out</sub>



#### (b)

**Figure 8:** Calculated and measured dependences of the module of impedance of inductors with the powder (a) and ferrite (b) cores on frequency

of the examined converter on the load resistance  $R_0$  at the fixed value of the duty factor of the control signal d = 0.5 at two values of the frequency of the control signal equal in turn 50 kHz and 400 kHz, are presented. Results of calculations passed with the use of the electrothermal model of the inductor are marked with solid lines, whereas results of calculations obtained by means of the linear model of the inductor are marked with dashed lines.



**Figure 9:** Calculated and measured dependences of the output voltage of the boost converter on the load resistance

As one can notice, the use of the electrothermal model of the inductor makes possible to obtain the considerably better agreement between performance of calculations and measurement than with the use of the linear model of the inductor. It is proper to notice that the regard of losses in the inductor and dependences of the inductances on frequency causes a decreasing in the output voltage of the considered converter. The use of the linear model of the inductor can cause the overestimate of results of calculations even about 50%.

In turn, Fig.10 illustrates the dependence of the core temperature  $T_{R}$  (solid lines) and the winding temperature  $T_{U}$  (dashed lines) on the load resistance corresponding to characteristics from Fig.9. As it is visible, for both considered frequencies the decreasing dependences  $T_{R}(R_{0})$  and  $T_{U}(R_{0})$  are obtained, whereas an increase in frequency causes a decrease in value of the temperature of the inductor. From the fact, that the winding temperature is lower than the core temperature results, that a main source of losses is the core of the inductor.



**Figure 10:** Calculated and measured dependences of the core and winding temperatures of the load resistance

#### 5 Conclusions

This paper describes the electrothermal model of the choking-coil with ferromagnetic the core dedicated for SPICE software and proposes a method of estimating values of magnetic, electrical and thermal parameters of this model. The proposed algorithm is simple to implement and largely uses the data presented by manufacturers of the ferromagnetic core and winding wire in the catalog data.

The investigations were performed for two arbitrarily chosen inductors with the core made of powdered iron and ferrite material. The presented experimental results show that the proposed method of estimating the parameters is correct, which is proved by good agreement between the measured and calculated characteristics of the considered inductors.

The electrothermal model of the inductor together with the proposed estimation method of its parameters can be useful for designers of switch-mode power supplies and in the analysis of the considered class of electronic circuits.

#### 6 Acknowledgements

This project is financed from the funds of National Science Centre which were awarded on the basis of the decision number DEC-2011/01/B/ST7/06738.

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Arrived: 06. 06. 2014 Accepted: 28. 10. 2014